

1. Backpropagation

Example 1

$$f(x, y, z) = z + \sin(x^2 + y \times \exp(z))$$

$$a = x^2 \quad d = a + c$$

$$b = \exp(z) \quad e = \sin(d)$$

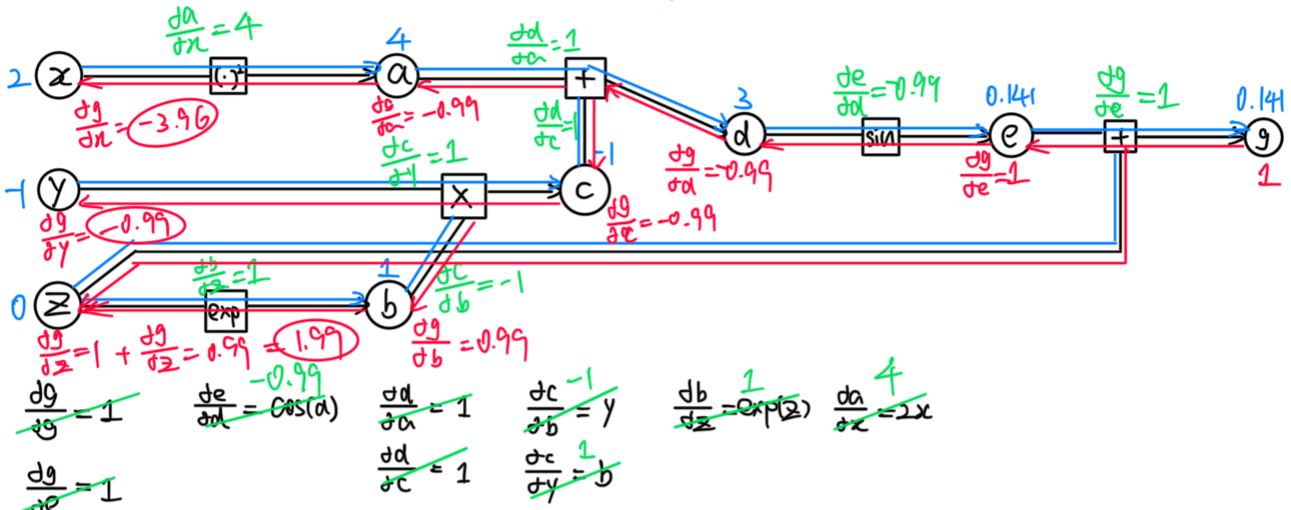
$$c = y \times b \quad g = z + e$$

$$x=2, y=-1, z=0$$

blue: forward pass

red: backward pass

green: local gradients



Example 2

$$f(x, y, z, v) = \exp(y - x - \log(z)) + (y - z)^2 \cdot \log(v)$$

$$= \exp(\exp(\log(y) \cdot (-x)) - \log(z)) + (y - z)^2 \cdot \log(v)$$

$$x=0, y=1, z=2, v=3$$

$$a = -x \quad e = \log(z) \quad i = h^2$$

$$b = \log(y) \quad f = d - e \quad j = \log(v)$$

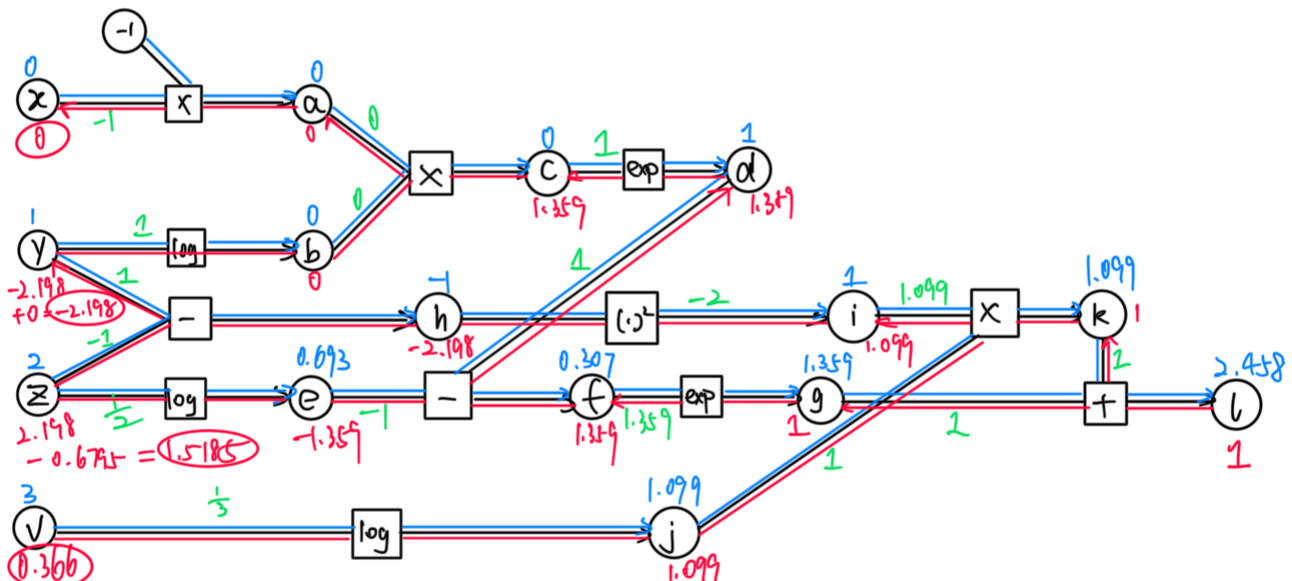
$$c = b \cdot a \quad g = \exp(f) \quad k = i \cdot j$$

$$d = \exp(c) \quad h = y - z \quad l = g + k$$

blue: forward pass

red: backward pass

green: local gradient



$$\begin{array}{cccccccc}
 \frac{\partial l}{\partial g} = 1 & \frac{\partial k}{\partial i} = j & \frac{\partial j}{\partial v} = \frac{1}{v} & \frac{\partial h}{\partial y} = 1 & \frac{\partial g}{\partial f} = \exp(f) & \frac{\partial f}{\partial e} = -1 & \frac{\partial d}{\partial c} = \exp(c) & \frac{\partial c}{\partial a} = b & \frac{\partial a}{\partial x} = -1 \\
 \frac{\partial l}{\partial k} = 1 & \frac{\partial k}{\partial j} = i & \frac{\partial i}{\partial h} = \frac{-2}{2h} & \frac{\partial h}{\partial z} = -1 & \frac{\partial f}{\partial u} = 1 & \frac{\partial e}{\partial z} = \frac{1}{z} & \frac{\partial c}{\partial b} = a & \frac{\partial b}{\partial y} = \frac{1}{y}
 \end{array}$$

Common derivatives & Operations

Common derivatives

$f(x)$	$f'(x)$
c	0
x	1
e^x	e^x
$\ln(x)$	$\frac{1}{x}$
Sigmoid $\sigma(x)$	$\sigma(x)(1-\sigma(x))$
$\tanh(x)$	$1 - \tanh^2(x)$
$\sin(x)$	$\cos(x)$
$\cos(x)$	$-\sin(x)$
$\tan(x)$	$\sec^2(x) = \frac{1}{\cos^2(x)}$

Chain Rule: $y = f(g(x))$, let $u = g(x)$, then

$$\frac{dy}{dx} = \frac{dy}{du} \cdot \frac{du}{dx} \quad * \text{Key in Backpropagation}$$

Key values in log and exp

$$\exp(0) = 1, \exp(1) \approx 2.718, e^{-\infty} \rightarrow 0$$

$$\ln(1) = 0, \ln(e) = 1, \ln(0) \text{ undefined.}$$

Key operations in log and exp

Multiplication $e^x \cdot e^y = e^{x+y}$

Division $\frac{e^x}{e^y} = e^{x-y}$

Exponentiation $(e^x)^y = e^{xy}$

Inversion $e^{\ln(x)} = x$

$$\ln(x \cdot y) = \ln(x) + \ln(y)$$

$$\ln\left(\frac{x}{y}\right) = \ln(x) - \ln(y)$$

$$\ln(x^y) = y \cdot \ln(x)$$

$$\ln(e^x) = x$$